High Order Hybrid Hermite-Discontinuous Galerkin Overset Grid Methods for the Wave Equation

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Introduction

Topic

Introducing a new high order method for the wave equation in complex geometry.

Methods

Hermite methods

- High computation to communication ratio.
- Stable with order independent CFL cond.
- Minimal stencil.
- Efficient on Cartesian grids.

DG methods

- Geometrically flexible.
- Boundary conditions are easy to enforce.

Hybrid Hermite-DG overset grid method

- All of above!
- Stable without any dissipation for a hyperbolic problem.

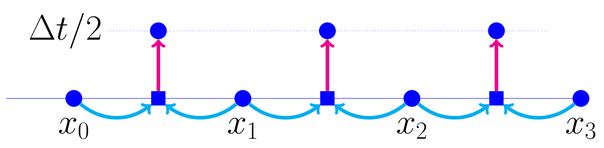
D. Appelö, T. Hagstrom and A. Vargas, *Hermite Methods for the Scalar Wave Equation*, SIAM J. Sci. Comput.
D. Appelö and T. Hagstrom, *A new discontinuous Galerkin formulation for wave equations in second order form. SINUM*, 53(6):2705-2765, 2015

Hermite methods

Algorithm

In Hermite methods the evolution is carried out on each cell $[x_i, x_{i+1}]$ in two stages:

- Hermite interpolation.
- Time evolution based on Taylor series and collocation of the PDE.



Step 1: Hermite interpolation

Construct Hermite interpolants

$$p_{i+\frac{1}{2}}(x) = \sum_{l=0}^{2m+3} \hat{u}(l) \left(\frac{x - x_{i+\frac{1}{2}}}{h} \right)^l,$$

satisfying

$$\frac{d^l p_{i+\frac{1}{2}}}{dx^l} = \left. \frac{d^l p_i}{dx^l} \right|_{x=x_i}, \quad \frac{d^l p_{i+\frac{1}{2}}}{dx^l} = \left. \frac{d^l p_{i+1}}{dx^l} \right|_{x=x_{i+1}}.$$

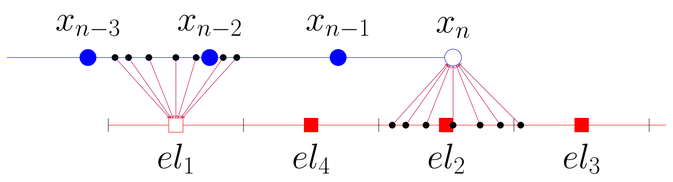
Step 2: Time evolution

Recursion relations for Taylor coefficients

$$\hat{u}(l, s+1) = \frac{\Delta t}{s} \hat{v}(l, s),$$

$$\hat{v}(l, s+1) = c^2 \frac{(l+1)(l+2)}{h^2} \frac{\Delta t}{s} \hat{u}(l+2, s).$$

Communication



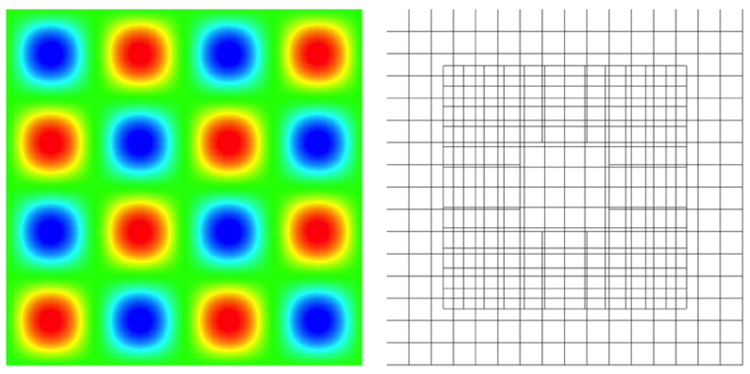
1: Evaluate Hermite/DG polynomial on quadrature points.

2: Compute the unknown DoF by L_2-projection.

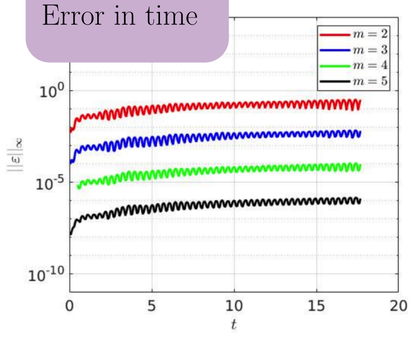
Syncronization

- DG timestep can be much smaller then Hermite timestep.
- Data communicated to a DG grid every Hermite timestep by evaluating Hermite space-time interpolants at each local DG time.

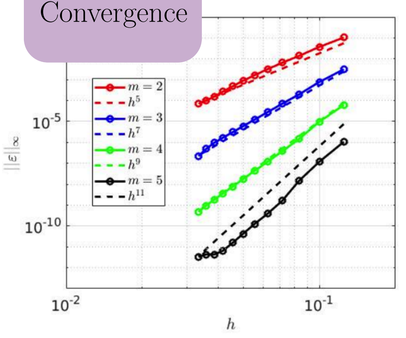
Stability and convergence



Error in time

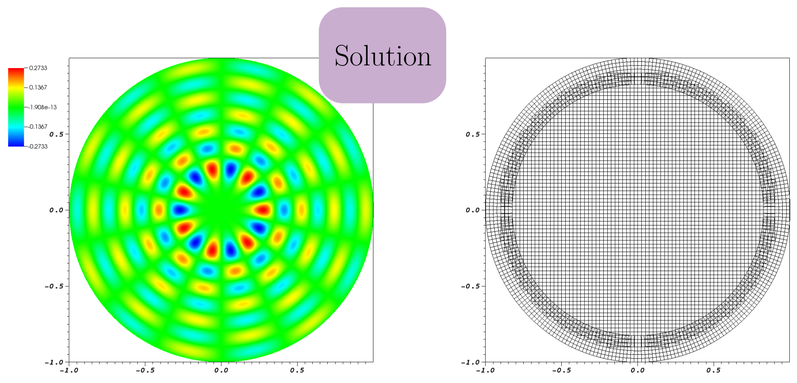


Convergence

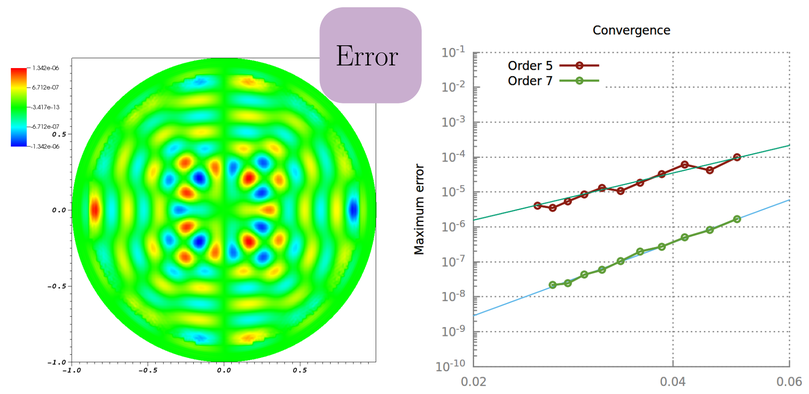


Experiment with a standing wave solution

Solution

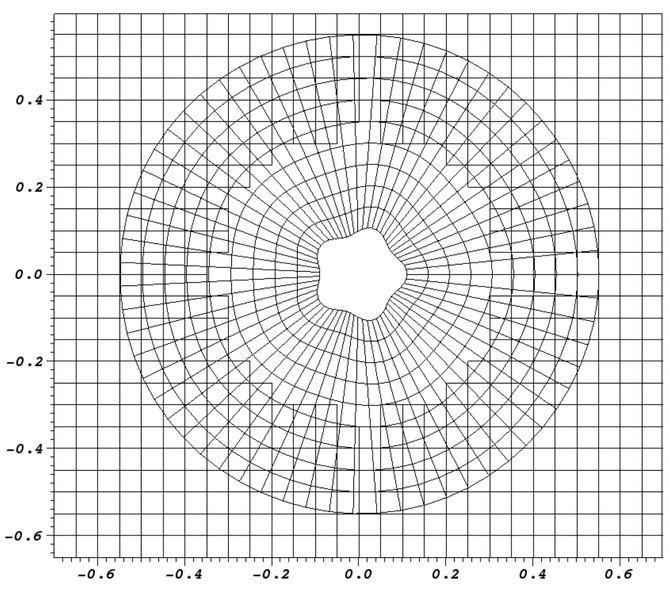


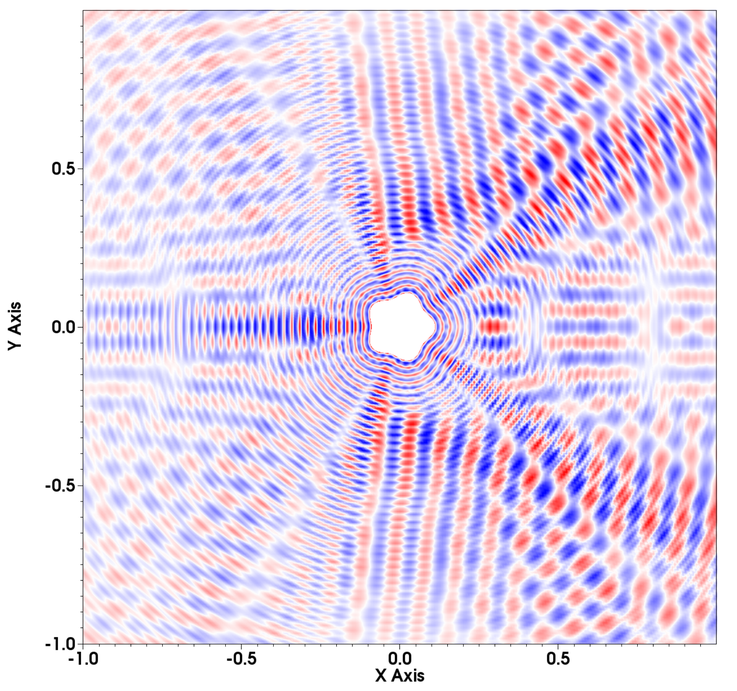
Error

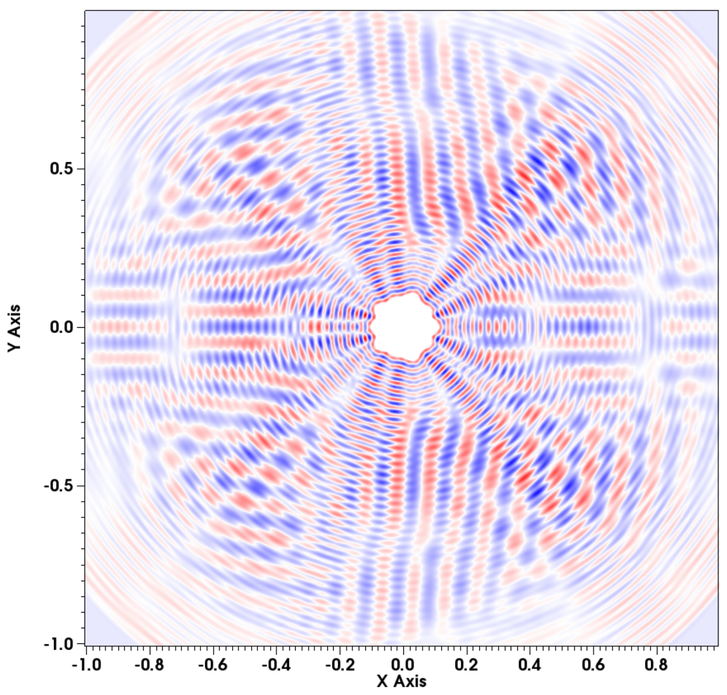


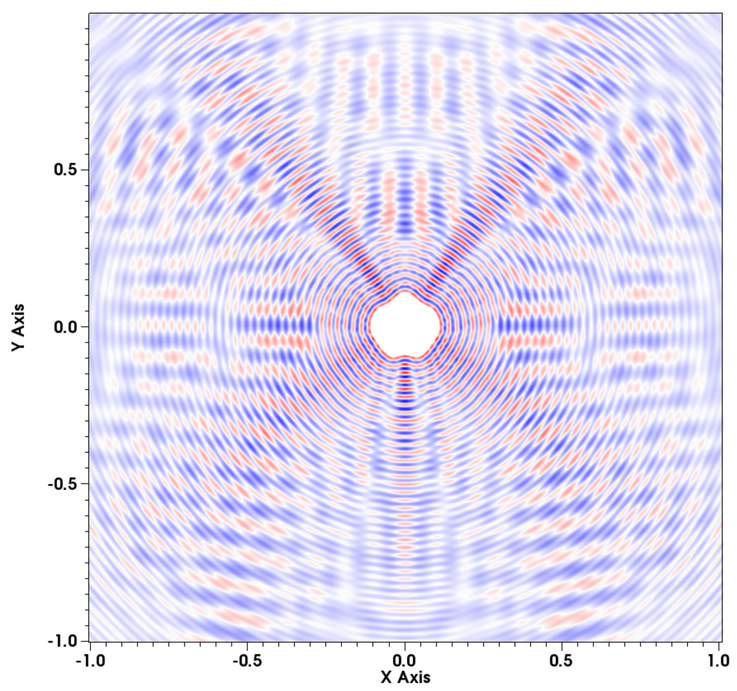
Experiment with complex geometry and PML

Scattering from a smooth pentagonal









LaTeX TikZposter