Overlapping Schwarz-Spectral Element Methods For Massively Parallel Flow

Ketan Mittal[1], S. Dutta[2], P. Fischer[1]
[1]University of Illinois Urbana-Champaign, [2]CSI, City University of New York

Scalable implementation of High-Order SEM based overset grid method enables high fidelity turbulent flow and heat transfer calculations in complex domains on leadership-class HPCs.

Motivation

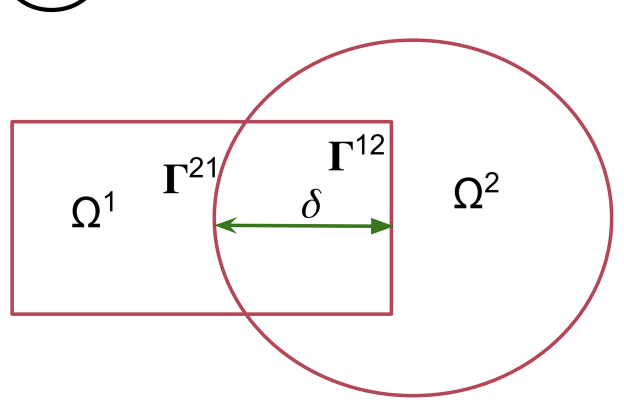
Overlapping grids can circumvent meshing problems posed by:

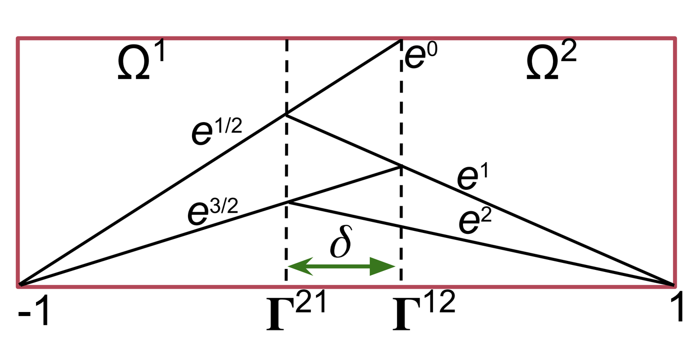
- Complicated geometries
- Problems featuring disparate length scales

Real world problems featuring slowly evolving temporal modes demand scalable methods.

Methodology

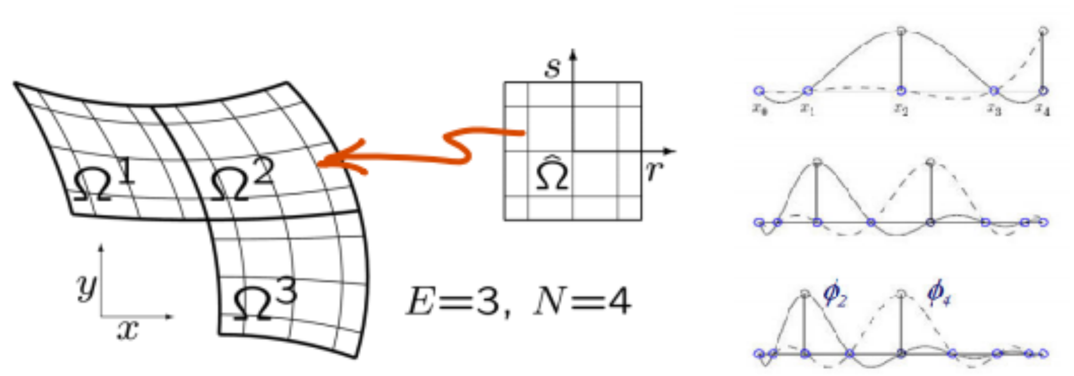
① Schwarz [1870]





② SEM [Patera' 84]

High-order weighted residual method featuring Legendre polynomials on Gauss-Lobatto-Legendre points

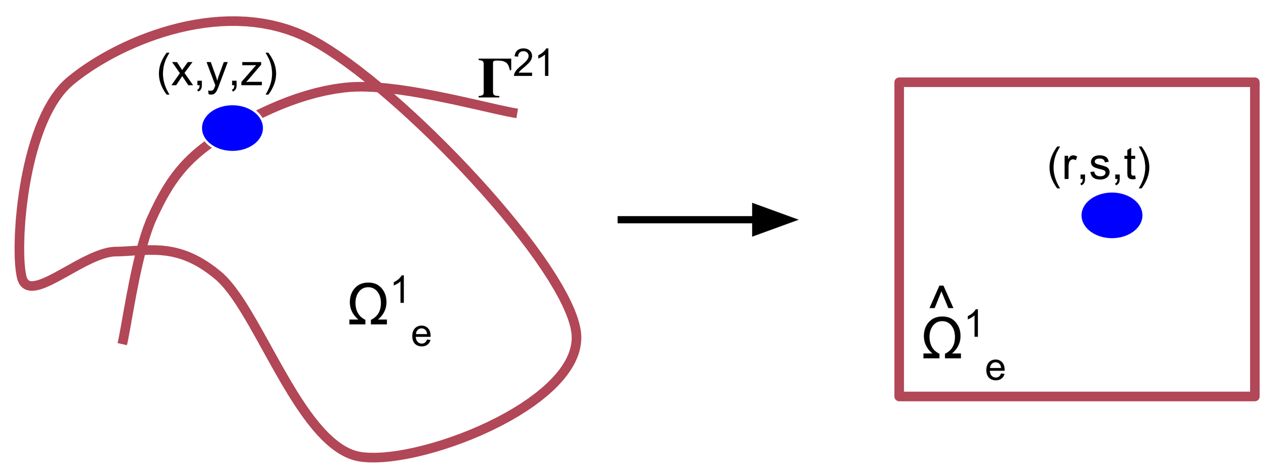


Incompressible Navier-Stokes + Schwarz Iterations [Merrill et. al. JCP'16]

$$\left.\begin{array}{c}\frac{1}{\Delta t}\beta_{0k}\mathbf{u}^n - \frac{1}{Re}\nabla^2\mathbf{u}^n + \nabla p^n = \mathbf{f}\\ \nabla.\mathbf{u}^n = 0\end{array}\right\} K_{max} \text{ iterations}$$

$$\left.\begin{array}{c}\mathbf{u}^{[i],k=1,n}(\mathbf{x}) = \sum_{p=1}^{\mathrm{IEXT}m}\gamma_p\mathbf{u}_{int}^{[j],k_{max},n-p}(\mathbf{x}),\mathbf{x}\in\Gamma^{ij}\\ \mathbf{u}^{[i],k>1,n}(\mathbf{x}) = \mathbf{u}_{int}^{[j],k-1,n-p}(\mathbf{x}),\mathbf{x}\in\Gamma^{ij}\end{array}\right\} \text{Extrapolated BC}$$

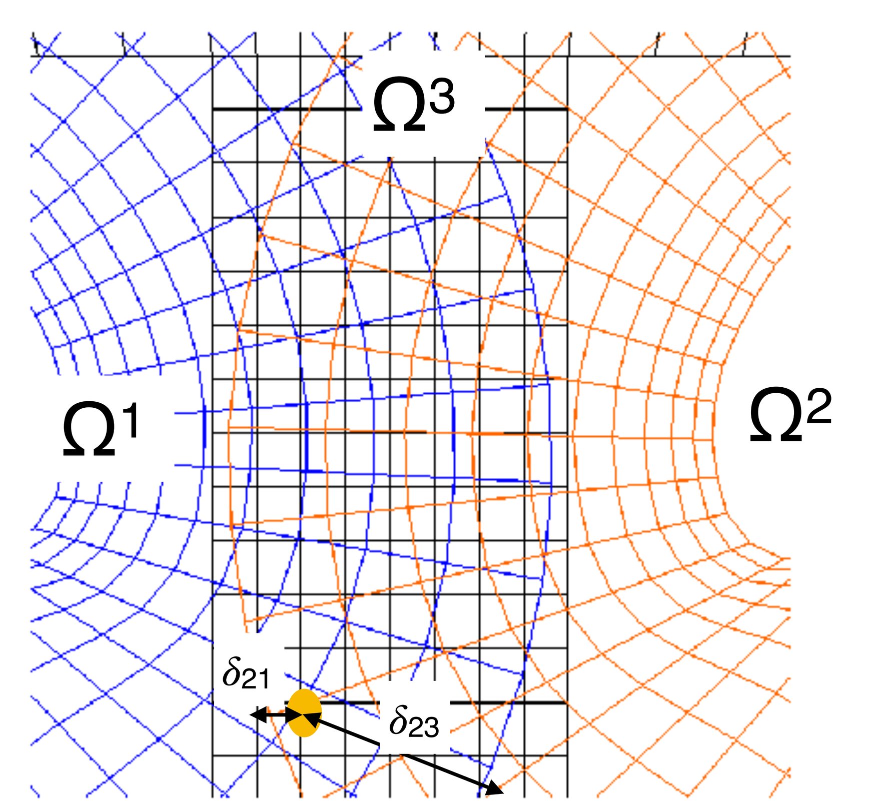
③ Fast spectral interpolation [*gslib*]



findpts: *(x,y,z) -> (r,s,t),p,e*
findpts_eval: *u,(r,s,t),p,e -> u**

- *Executes with at-most* $log_2 P$ complexity
- Obviates need for any MPI code

④ Multi-Domain calculations

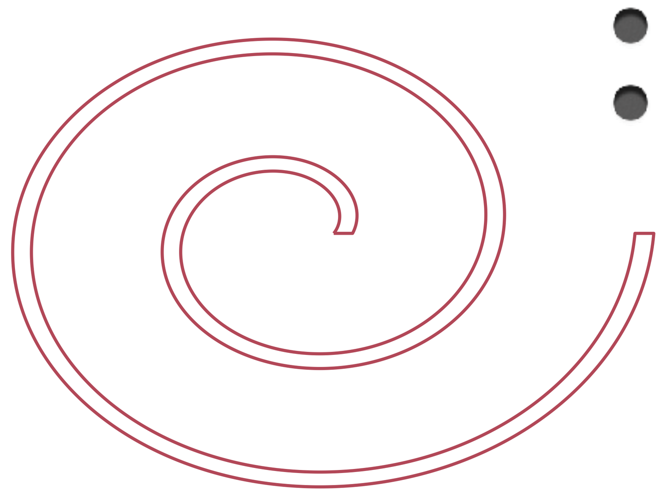


Discriminators:

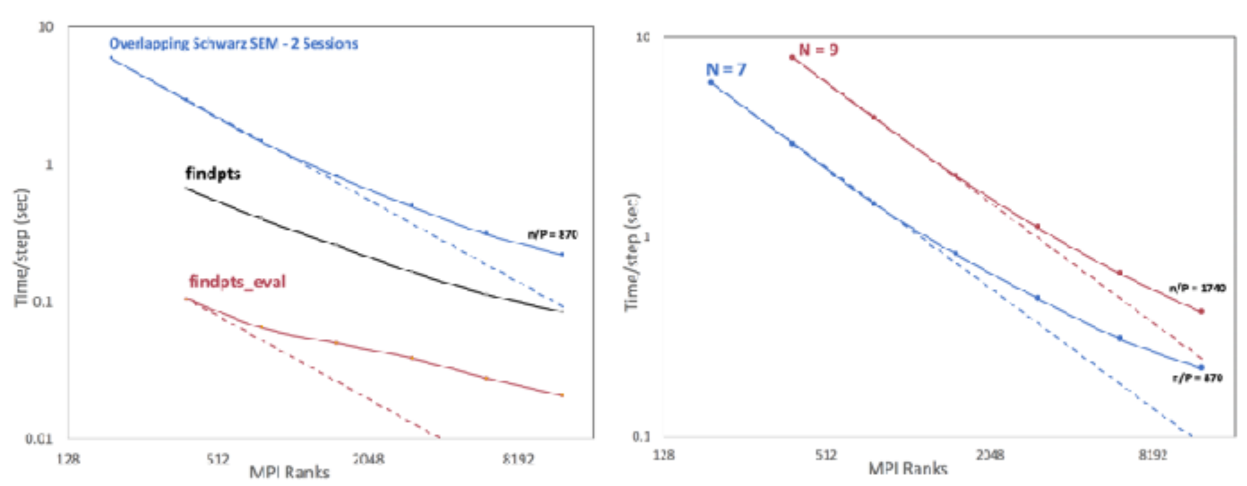
- Mesh ID
- Distance from interface

Results

findpts



- 20th Order Element
- *gslib-findpts* finds 100 points in 0.002 s with machine precision

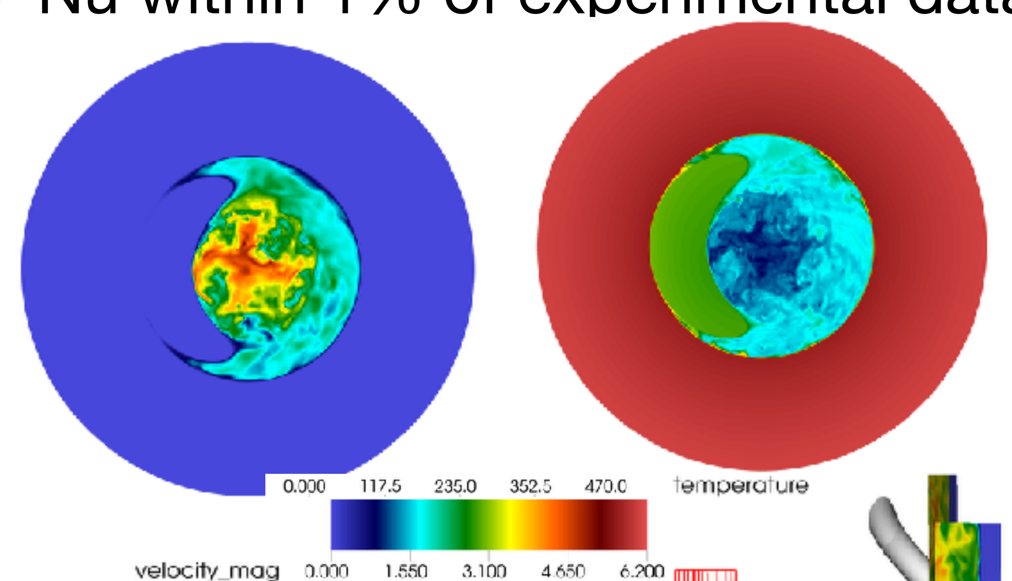


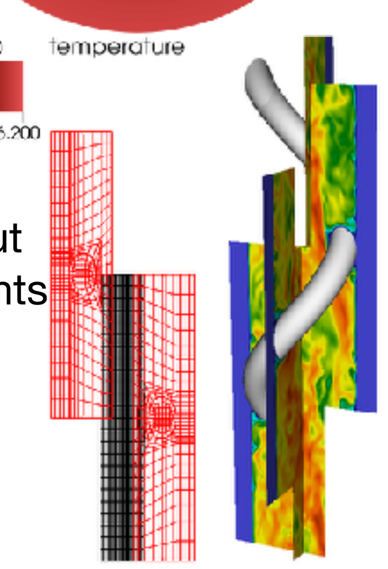
Scaling

- *20,000* spectral elements (*N=7*)
- *2000* elements at interface (*128,000* grid points)
- Scaling meets theoretical expectations [Fischer' 15]

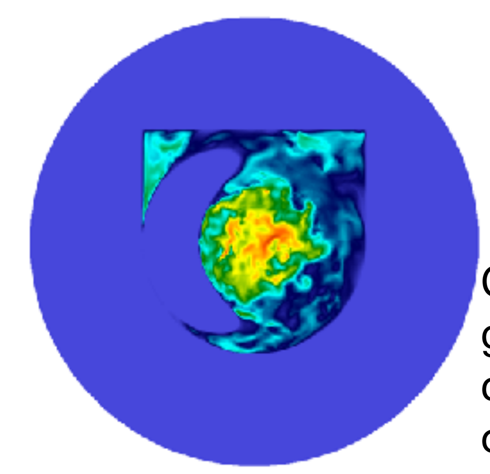
Heat transfer in a pipe with wire-coil

- *Re* = 5300
- Fixed flow rate through a coupled unsteady Stokes solve
- High quality computational mesh
- Nu within 1% of experimental data





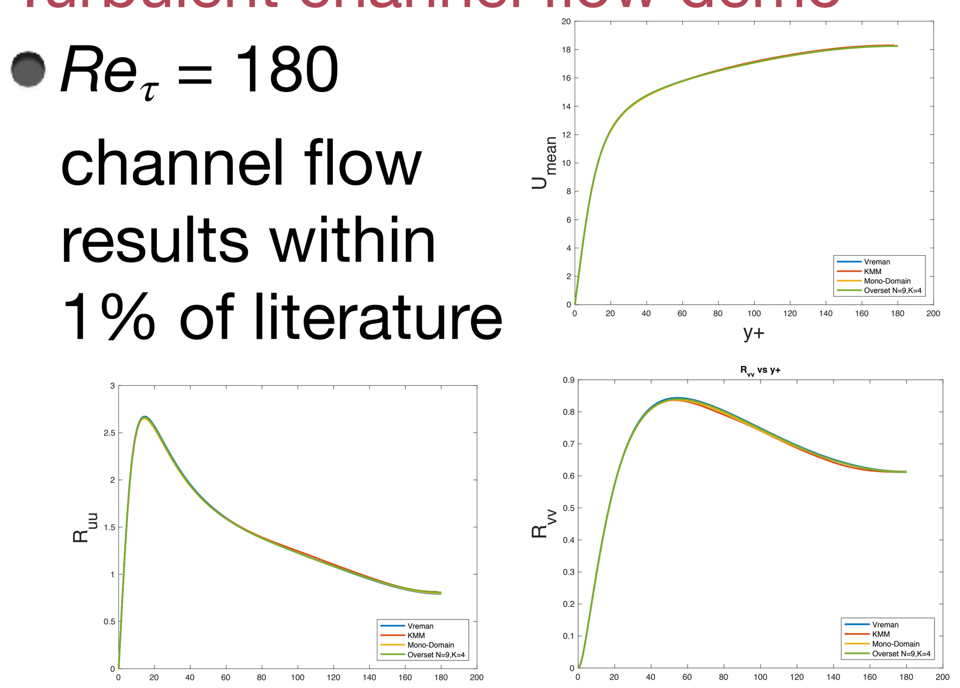
Coil in circular-casing can be done without overset grids, but leads to highly skewed elements



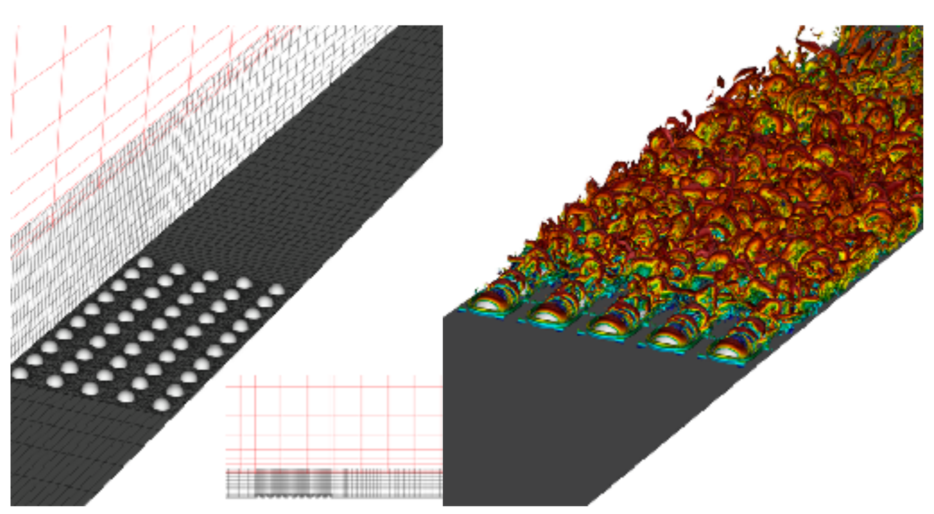
Coil in U-slot casing: Mesh generation is extremely complicated without overlapping grids

Turbulent channel flow demo

- Re_τ = 180 channel flow results within 1% of literature



- Flow over roughness at *Re* = 5,000
- 80% reduction in element count in upper-zone



Impact

- *findpts* obviates need for any MPI code
- Recent developments allow multiple overlapping meshes
- Heat transfer analysis of coil in a pipe show excellent comparison with experimental data
- Turbulent flow calculations show good agreement in first and second order statistics