A stable added-mass partitioned (AMP) scheme for elastic solids and incompressible flows.

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New FSI scheme for INS fluids and elastic solids

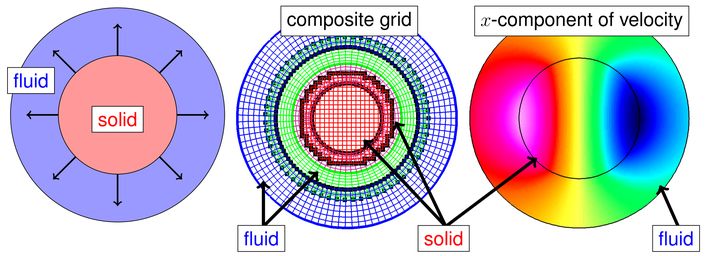
Goal:
Develop a robust non-iterative partitioned scheme for fluid-structure interaction (FSI).

Approach:

- Enforce Robin interface conditions motivated from the solid characteristic structure.
- Advance the fluid velocity and pressure using a fractional-step scheme.
- Tackle complex moving geometries using deforming composite grids (DCGs).

Results:
Algorithm is stable and second-order accurate for all mass ratios of the fluid and solid, even for light solids when added mass effects are large.

Governing equations for the FSI problem



Fluid equations:

$$\rho\partial_t\mathbf{v} + \rho(\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla p + \mu\Delta\mathbf{v}, \quad \mathbf{x}\in\Omega_f$$
$$\Delta p = -\rho\nabla\mathbf{v} : (\nabla\mathbf{v})^T, \quad \mathbf{x}\in\Omega_f$$
$$\nabla\cdot\mathbf{v} = 0, \quad \mathbf{x}\in\partial\Omega_f$$

Solid equations:

$$\partial_t\bar{\mathbf{u}} = \bar{\mathbf{v}}, \quad \mathbf{x}\in\Omega_s$$
$$\bar{\rho}\partial_t\bar{\mathbf{v}} = \nabla\cdot\bar{\boldsymbol{\sigma}}, \quad \mathbf{x}\in\Omega_s$$

Stress tensors:

$$\boldsymbol{\sigma} = -p\mathbf{I} + \boldsymbol{\tau}, \quad \boldsymbol{\tau} = \mu\left(\nabla\mathbf{v} + (\nabla\mathbf{v})^T\right), \quad \bar{\boldsymbol{\sigma}} = \bar{\lambda}(\nabla\cdot\bar{\mathbf{u}}) + \bar{\mu}\left(\nabla\bar{\mathbf{u}} + (\nabla\bar{\mathbf{u}})^T\right)$$

Interface conditions:

$$\mathbf{v} = \bar{\mathbf{v}}$$
$$\boldsymbol{\sigma}\mathbf{n} = \bar{\boldsymbol{\sigma}}\mathbf{n}$$

$(\rho, \bar{\rho})$ – fluid and solid densities
μ – fluid dynamic viscosity
$(\bar{\lambda}, \bar{\mu})$ – solid Lamé parameters
$\mathbf{n}$ – outward fluid normal

The AMP interface conditions are derived from solid characteristics

Traditional scheme:

$\mathbf{v} = \bar{\mathbf{v}}^{(p)}$
$\bar{\boldsymbol{\sigma}}\mathbf{n} = \boldsymbol{\sigma}^{(p)}\mathbf{n}$

Heavy solids / Coarse grids

AMP scheme

Light solids / Fine grids

Anti-traditional scheme:

$\bar{\mathbf{v}} = \mathbf{v}^{(p)}$
$\boldsymbol{\sigma}\mathbf{n} = \bar{\boldsymbol{\sigma}}^{(p)}\mathbf{n}$

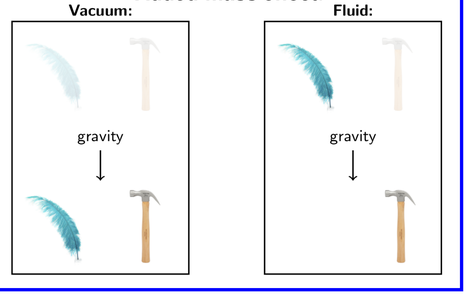
fluid sees solid as no-slip wall
solid sees fluid as a free surface

solid sees fluid as no-slip wall
fluid sees solid as a free surface

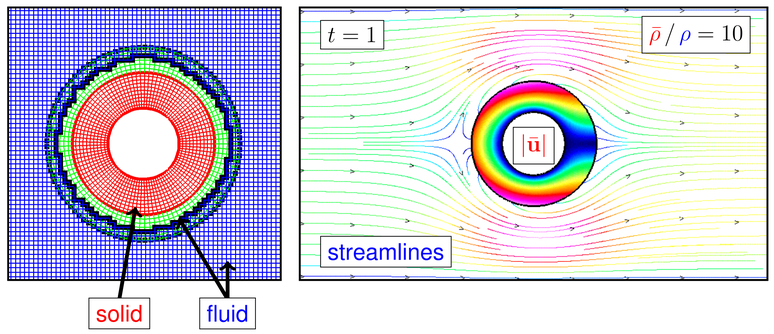
The AMP scheme approaches the traditional and anti-traditional schemes in the light and heavy solid limits, respectively.

- The traditional scheme is **sensitive** to numerical instabilities when added mass effects are large.
- The AMP scheme is **insensitive** to added-mass instabilities.

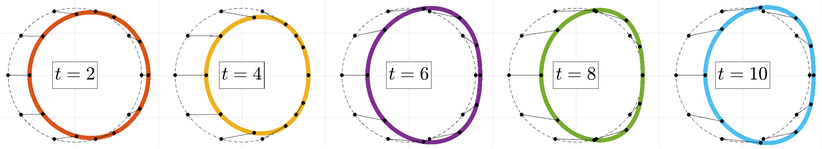
Added-mass effect:



Results for a benchmark problem: elastic annulus in a channel



Various snapshots show the time evolution of the solid interface.

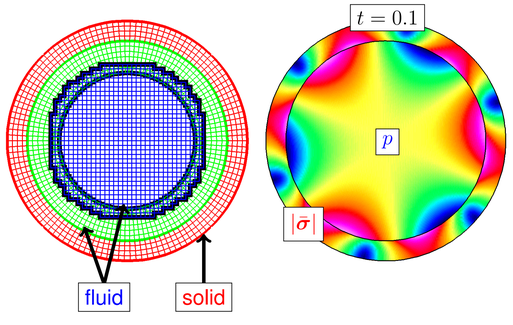


(displacements are magnified by 20 to accentuate the deformation).

Max-norm self-convergence rates are computed and are close to **second-order**.

h_j	$E_j^{(p)}$	r	$E_j^{(\mathbf{v})}$	r	$E_j^{(\bar{\mathbf{u}})}$	r	$E_j^{(\bar{\mathbf{v}})}$	r	$E_j^{(\bar{\boldsymbol{\sigma}})}$	r
1/20	6.3e-3		7.4e-3		1.7e-4		4.1e-4		1.3e-2	
1/40	1.7e-3	3.7	1.5e-3	5.0	4.6e-5	3.7	9.2e-5	4.5	3.7e-3	3.6
1/80	4.7e-4	3.6	3.0e-4	4.9	1.2e-5	3.7	2.0e-5	4.5	1.0e-3	3.6
rate	**1.86**		**2.30**		**1.90**		**2.18**		**1.86**	

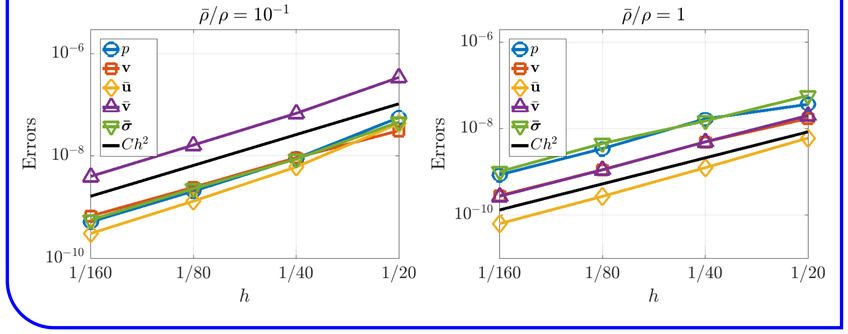
Exact solutions are derived to verify second-order accuracy



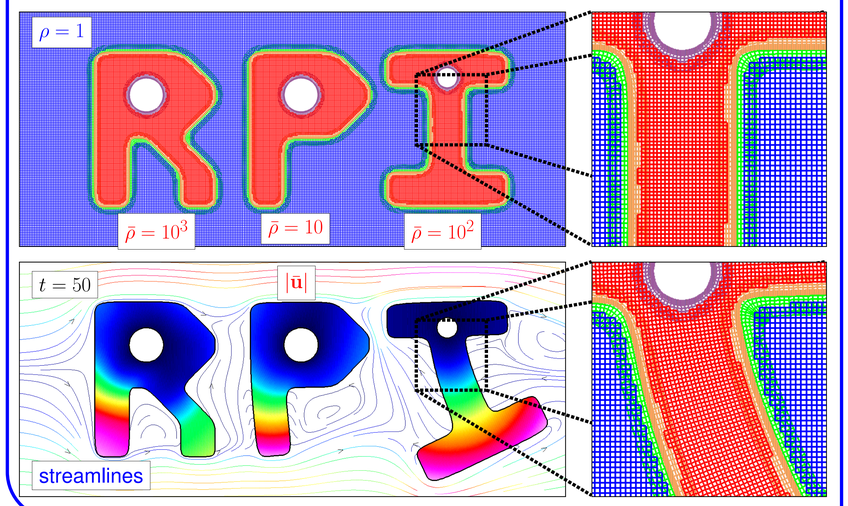
Radial traveling wave solution:

$$\mathbf{v}(r,\theta,t) = \hat{\mathbf{v}}(r)e^{i(n\theta-\omega t)},$$
$$p(r,\theta,t) = \hat{p}(r)e^{i(n\theta-\omega t)},$$
$$\bar{\mathbf{u}}(r,\theta,t) = \hat{\bar{\mathbf{u}}}(r)e^{i(n\theta-\omega t)}$$

The numerical solution converges close to **second-order accuracy**.



Deforming composite grids are used for complex geometries



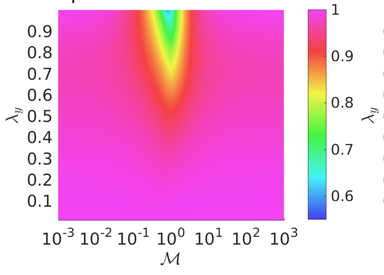
Normal-mode analysis confirms stability for the AMP scheme

- The AMP scheme is stable for all possible mass ratios.
- The traditional scheme will eventually become unstable as the grid is refined.

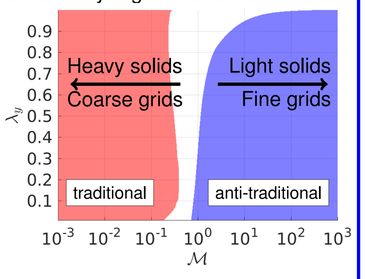
Second-order accurate discretization of inviscid model problem:

$$\mathcal{M} = \frac{\rho H}{\bar{\rho}\Delta y}$$
$$\lambda_y = \frac{\bar{c}_p\Delta t}{\Delta y}$$

max amplification factor for AMP scheme



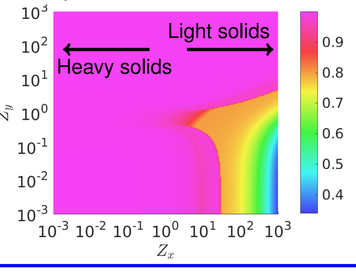
stability regions for other schemes



First-order accurate discretization of viscous model problem:

$$Z_x = \frac{\mu k_x}{\bar{\rho}\bar{c}}$$
$$Z_y = \frac{\mu/H}{\bar{\rho}\bar{c}}$$

max amplification factor for AMP scheme



Conclusion

- Deforming composite grids are used to resolve complex geometries.
- Various examples demonstrate robustness and second-order accuracy.
- Stability can be proven for various model problems.

J. W. Banks, W. D. Henshaw, D. W. Schwendeman, An analysis of a new stable partitioned algorithm for FSI problems. Part I: Incompressible flow and elastic solids, J. Comput. Phys. 269 (2014) 108-137.