# Solving Fluid-Structure Interaction Problems using Overture

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14th Symposium on Overset Composite Grids and Solution Technology,
University of Maryland, College Park, Maryland, October 1–4, 2018.

# Acknowledgments.

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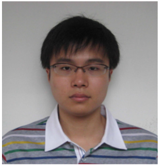


Qi Tang
(LANL)

**RPI graduate students:**
Jordan Angel (now at NASA Ames), John Jacangelo, Kamala Liu,
Fanlong Meng (now at COMSOL), Dan Serino

**Supported by:**
National Science Foundation, RPI, LLNL.
Department of Energy, Office of Science, ASCR Applied Math Program.

# Outline: Fluid-Structure Interaction (FSI) Problems

Current focus: partitioned algorithms that are stable for light solids.

This talk will give an overview of some of our recent FSI work.

We have been working in a number of areas:

1. incompressible flow and bulk elastic solids (Dan Serino's poster)
2. incompressible flow and rigid bodies (Qi Tang's talk)
3. incompressible flow and beams (Longfei Li's talk)
4. conjugate heat transfer
5. fourth-order accurate BDF-IMEX scheme for incompressible flows
6. fourth-order accurate multigrid algorithms (Kamala Liu's poster)

# Overture and the CG solvers source code

The Overture framework and Composite Grid (CG) PDE solvers are open source and available from (documentation, downloads)

`overtureFramework.org`

The source-code repositories are hosted at

`sourceforge.net/projects/overtureframework`

# Monolithic vs Partitioned Schemes

**Monolithic FSI Solver**

Fluid + solid PDEs

Interface conditions:

$$\boxed{\begin{aligned} D &: [v] = 0 \\ N &: [\mathbf{n}^T \boldsymbol{\sigma}] = 0 \end{aligned}}$$

**Partitioned FSI Solver**

**Solid Solver**

Solid PDE

Solid BC : ?

**Fluid Solver**

Fluid PDE

Fluid BC : ?

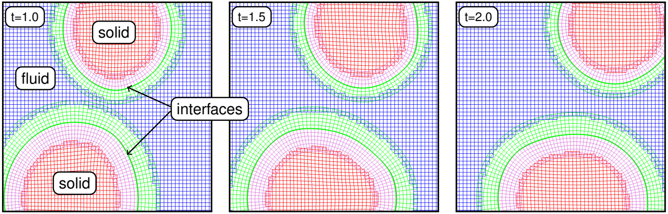
Objective: Choose the solid BC and fluid BC so the partitioned scheme is stable and accurate.

# Fluid Structure Interaction (FSI) algorithms

Our approach in Overture is based on moving and deforming composite grids.

Approach:

- Fluids are solved in an Eulerian frame.
- Solids are solved in a Lagrangian frame.
- Grids around rigid bodies are moved at each time step.
- Deforming interface grids are regenerated at each time-step with the hyperbolic grid generator.



# Basic FSI Algorithm.

**procedure** $\textsc{solveFsiDcg}(\mathcal{G}, t_{\text{final}})$ $\triangleright$ Input: initial composite grid and final time

    $t := 0;\; n := 0;\; \mathcal{G}^n = \mathcal{G};$

    $\texttt{assignInitialConditions}(\mathbf{q}_\mathbf{i}^n, \bar{\mathbf{q}}_\mathbf{i}^n, \mathcal{G}^n);$

    **while** $t < t_{\text{final}}$ **do**

        $\Delta t := \texttt{computeTimeStep}(\mathbf{q}_\mathbf{i}^n, \bar{\mathbf{q}}_\mathbf{i}^n, \mathcal{G}^n);$

        $\mathcal{G}^p := \texttt{moveGrids}(\mathcal{G}^n, \mathbf{q}_\mathbf{i}^n, \bar{\mathbf{q}}_\mathbf{i}^n);$ $\triangleright$ (calls HyperbolicMapping)

        $\mathcal{G}^p := \texttt{updateOverlappingGrid}(\mathcal{G}^p);$ $\triangleright$ (Ogen)

        $\mathbf{q}_\mathbf{i}^{n+1} := \texttt{advanceFluid}(\mathbf{q}_\mathbf{i}^n, \mathcal{G}^n, \mathcal{G}^p, \Delta t);$

        $\bar{\mathbf{q}}_\mathbf{i}^{n+1} := \texttt{advanceSolid}(\bar{\mathbf{q}}_\mathbf{i}^n, \mathcal{G}, \Delta t);$

        $(\mathbf{n}^T\mathbf{v}^I, \mathbf{n}^T\boldsymbol{\sigma}^I) := \texttt{projectInterface}(\mathbf{q}_\mathbf{i}^{n+1}, \bar{\mathbf{q}}_\mathbf{i}^{n+1}, \mathcal{G}^p);$

        $\mathbf{q}_\mathbf{i}^{n+1} := \texttt{applyFluidBCs}(\mathbf{q}_\mathbf{i}^{n+1}, \mathcal{G}^p, \mathbf{n}^T\mathbf{v}^I, \mathbf{n}^T\boldsymbol{\sigma}^I);$

        $\bar{\mathbf{q}}_\mathbf{i}^{n+1} := \texttt{applySolidBCs}(\bar{\mathbf{q}}_\mathbf{i}^{n+1}, \mathcal{G}, \mathbf{n}^T\mathbf{v}^I, \mathbf{n}^T\boldsymbol{\sigma}^I);$

        $\mathcal{G}^{n+1} := \texttt{correctMovingGrids}(\mathbf{q}_\mathbf{i}^{n+1}, \bar{\mathbf{q}}_\mathbf{i}^{n+1}, \mathcal{G}^p, \Delta t);$

        $t := t + \Delta t;\quad n := n + 1;$

    **end while**

**end procedure**

# Deforming grid example

## Incompressible flow past a beam

beam grid: Click for video

# Deforming grid example

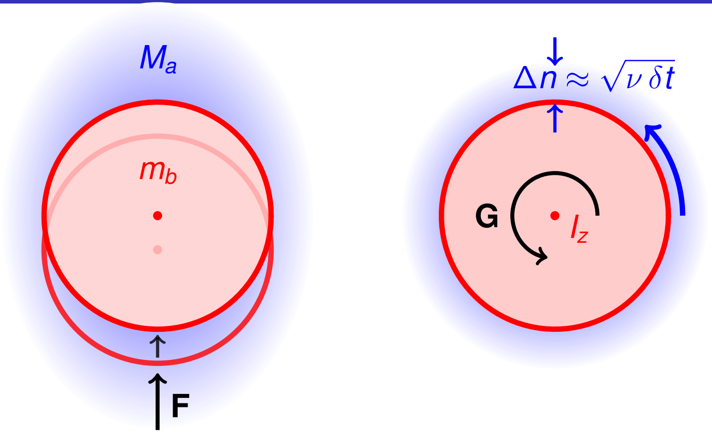
## Compressible flow past two elastic disks

multi-disk grid: Click for video

multi-disk grid: Click for video

# Added-mass and added-damping

## Partitioned schemes may be unstable when added-mass or added-damping is large




Added-mass: mass of fluid moved when a force is applied to the body
Added-damping: drag on the body when a force is applied (depends on $\delta t$)

# Movies: FSI for different regimes

## Fluid: compressible and incompressible; Solids: rigid, beams, bulk

Plate and particles: Euler + rigid-body: Click for video

Beam: Euler + nonlinear-beam-model: Click for video

Hele-shaw flow: Click for video
deforming cylinder: INS: Click for video

two rectangles: INS + rigid-body: Click for video

flexible channel: INS + beam-model: Click for video
39 particles: Euler + rigid-body: Click for video

Solid beam: Euler+nonlinear-solid: Click for video

Two beams: INS+Beam-model: Click for video
Ellipse: Euler+RB: Click for video

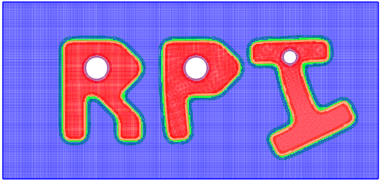
Stick: INS + Rigid-body: Click for video

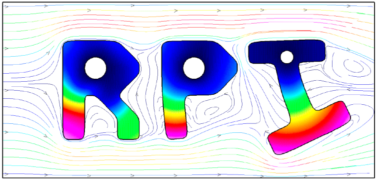
Drops: INS + Rigid-body: Click for video

# Incompressible flows and bulk solids

The AMP scheme for incompressible flow and bulk elastic solids has been extended to finite deformations (from the linearized model problem).

1. stable for any ratio of the fluid to solid densities.
2. interface conditions based on an impedance weighted average.
3. fractional-step IMEX scheme for the fluid with viscous terms implicit.
4. new scheme required modifications to the interface conditions from the original formulation to treat a subtle viscous instability.

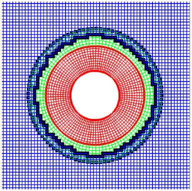


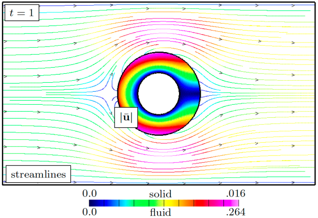


More details can be found in Dan Serino's poster.

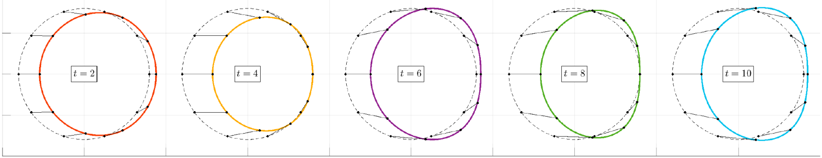
# Incompressible flows and bulk solids

## Benchmark: flow past an elastic solid annulus (fixed at the core).






Interface deformation (magnified), $\bar{\rho}/\rho = 10$




| Elastic annulus in a channel, $\delta = 10$ | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $h_j$ | $E_j^{(p)}$ | $r$ | $E_j^{(\mathbf{v})}$ | $r$ | $E_j^{(\bar{\mathbf{u}})}$ | $r$ | $E_j^{(\bar{\mathbf{v}})}$ | $r$ | $E_j^{(\bar{\boldsymbol{\sigma}})}$ | $r$ |
| 1/20 | 6.3e-3 | | 7.4e-3 | | 1.7e-4 | | 4.1e-4 | | 1.3e-2 | |
| 1/40 | 1.7e-3 | 3.7 | 1.5e-3 | 5.0 | 4.6e-5 | 3.7 | 9.2e-5 | 4.5 | 3.7e-3 | 3.6 |
| 1/80 | 4.7e-4 | 3.6 | 3.0e-4 | 4.9 | 1.2e-5 | 3.7 | 2.0e-5 | 4.5 | 1.0e-3 | 3.6 |
| rate | 1.86 | | 2.30 | | 1.90 | | 2.18 | | 1.86 | |

Self convergence tests show second-order accuracy.

# Incompressible flows and bulk solids

## These are very difficult for partitioned algorithms

The traditional partitioned (TP) coupling algorithm can be unstable even for relatively heavy bulk solids. Define the *density ratio* $\delta$:

$$\delta = \frac{\bar{\rho}}{\rho} = \frac{\text{solid density}}{\text{fluid density}}$$

Results for the annulus in a channel benchmark using the TP scheme:

| *h* | $\delta = 100$ | $\delta = 200$ | $\delta = 300$ | $\delta = 400$ | $\delta = 500$ | $\delta = 800$ | $\delta = 1000$ |
|---|---|---|---|---|---|---|---|
| 1/20 | unstable | unstable | stable | stable | stable | stable | stable |
| 1/40 | unstable | unstable | unstable | unstable | stable | stable | stable |
| 1/80 | unstable | unstable | unstable | unstable | unstable | unstable | stable |

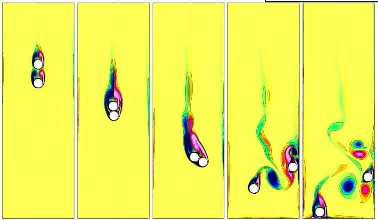
In agreement with the theory, the TP scheme becomes unstable, *no matter how heavy the solid*, once the grid becomes fine enough.

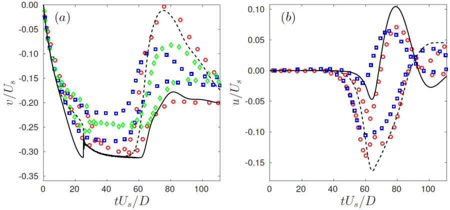
Movie: 5 bodies: INS + linear-elasticty: Click for video

# Validation of the algorithm for some rigid-body particulate flows.

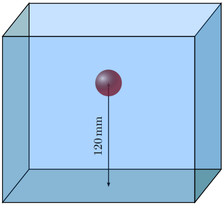
## Collaboration with Cambridge University, Schlumberger.

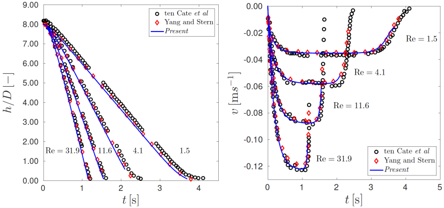
Drafting, kissing, tumbling cylinders






Falling sphere

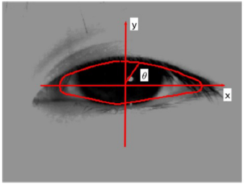


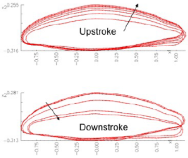



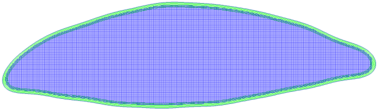

*Direct numerical simulation of particulate flows with an overset grid method*, A.R. Koblitz, S. Lovett, N. Nikiforakisa, WDH, JCP 2017.

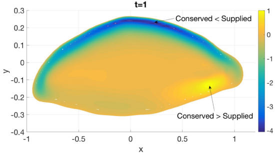
## Collaboration with Prof. Kara Maki, RIT, and colleagues.

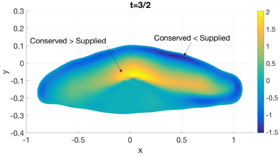
The thickness of the tear film is modeled as the nonlinear evolution equation $h_t + \nabla\cdot(\frac{h^3}{12}S\Delta h) = 0$. The eye motion for a partial blink is extracted from experiments.

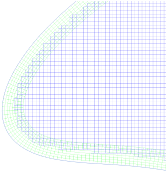




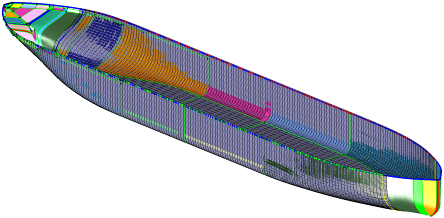



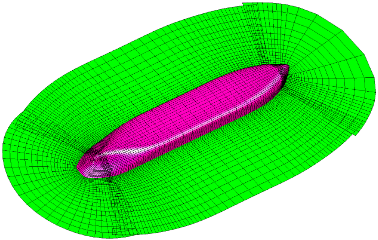



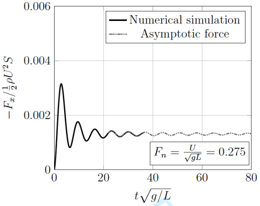
*A Model for Tear Film Dynamics During a Realistic Blink,* Kara L. Maki, WDH, Alex McManus, Richard J. Braun, Dylan M. Chapp, Tobin A. Driscoll, Journal for Modeling in Ophthalmology, to appear.

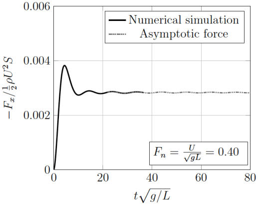
# 4th-order potential flow solver for ship wave resistance

## Collaboration with DTU (Technical University of Denmark)








*A stable, high-order Finite-difference method for estimating the wave resistance of ships*, Mostafa Amini-Afshar, Harry B. Bingham, WDH, in revision.

# Summary

1. Overture can be used to solve FSI problems in a variety of flow regimes.
2. Moving and deforming overset grids are used to treat the moving rigid or deformable solids.
3. Traditional partitioned schemes for FSI may suffer from *added-mass* or *added-damping* instabilities for light solids.
4. We have developed AMP schemes that over-come the added-mass instability for a variety of regimes and *require no sub-iterations*.

The Overture software is freely available from `overtureFramework.org`.